Perryfields Junior School
Calculation Policy 2025

This calculation policy aims to ensure consistency and continuity, with methods that build progressively from Year 3 to Year 6. For a detailed outline of key skill development across each year group, please refer to the Perryfields Maths Curriculum Overview and Progression documents.

It is essential that rapid recall of key number facts is embedded prior to written calculations being taught. This is necessary as the written calculations outlined in this policy rely on mental strategies to process numbers efficiently and with confidence. Therefore, mental strategies are included within this policy.

The links between subtraction, addition, division and multiplication are constantly reinforced throughout all year groups. These are particularly relevant when looking at number facts. The concept of the inverse operation will help the children develop the ability to complete mental calculations and the term will be introduced to children in Year 3. Children will also be encouraged to use this to check their calculations.

Children's understanding of place value is central to all of these calculation processes. Developing an understanding of numeracy, quantity and the number system is of intrinsic importance to the ability to be successful in calculation. Therefore, structured place value apparatus (e.g. Dienes, place value counters) are on the tables when children are working through a new calculation method to help them see this relationship and to develop their understanding of the processes they are working through. This will provide visual images and models of numbers and calculations which will enable children to develop a strong sense of numeracy.

As part of every lesson, emphasis will be made on mathematical vocabulary and children should have access to written vocabulary at all times to ensure they can recognise, spell and use it in their explanations.

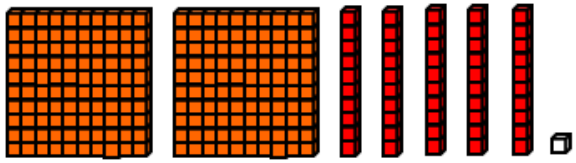
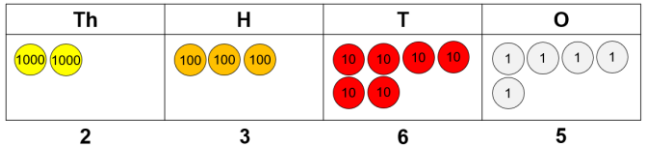
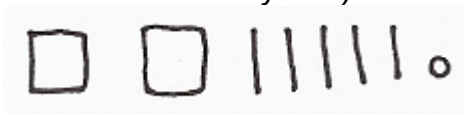
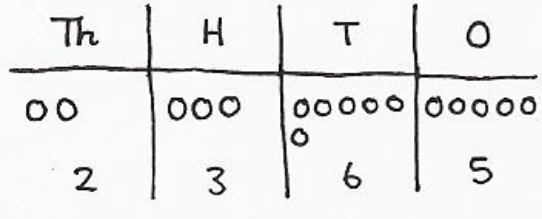
Contents:

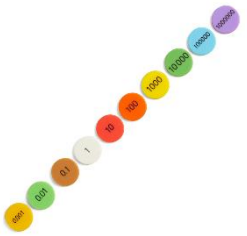
- Place value
- Addition
- Subtraction
- Multiplication
- Division
- Additional guidance: BIDMAS, fractions and the four operations, converting units of measurement
- Appendix: RUCSAC and presentation expectations

Calculation policy: place value

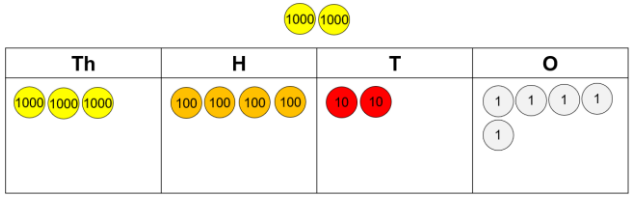
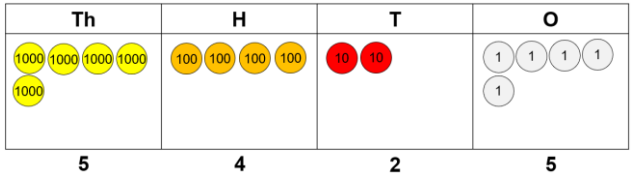
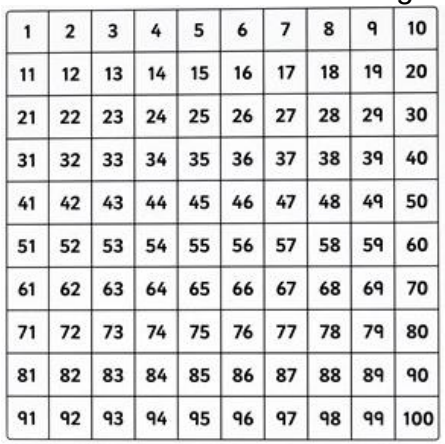
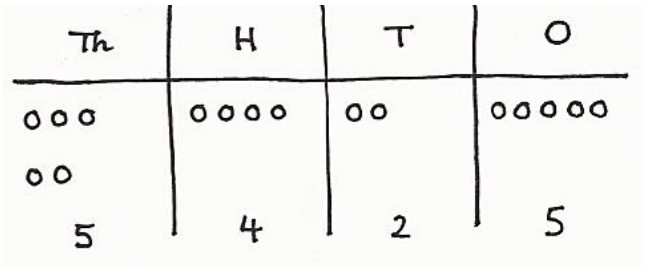
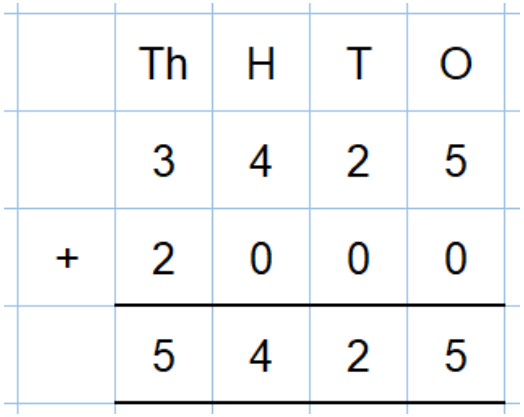
Class teachers will model reading and writing numbers using commas to demarcate thousands and millions (such as 12,450); however, children are not to write commas when writing numbers or calculations as to avoid looking like a decimal point (12450).

Place value

Concrete	Pictorial	Abstract
Representing numbers using Dienes with and without a place value chart (up to 4 digits): 251 (two hundred and fifty-one):  This should include examples with zeroes in different place value columns: for example, 105 (one hundred and five): **Place value counters (4 digits upwards):** 2365 (two thousand, three hundred and sixty-five): 	**Representing numbers with Dienes (up to 4 digits):** 251 (two hundred and fifty-one):  This should include examples with zeroes in different place value columns: for example, 105 (one hundred and five): **Place value counters (4 digits upwards):** 2365 (two thousand, three hundred and sixty-five): 	**Representing numbers using the abstract (digits under place value headings):** 251 (two hundred and fifty-one): H T O 2 5 1 2365 (two thousand, three hundred and sixty-five): Th H T O 2 3 6 5 One digit per square. Place value headings represented as follows: • O - Ones • T - Tens • H - Hundreds • Th - Thousands • TTh - Ten Thousands • HTh - Hundred Thousands • M - Millions

Children can make and compare numbers using Dienes (up to 4 digits) and place value counters (4 digits upwards).	Children can make and compare numbers by drawing pictorial representations using Dienes (up to 4 digits) and place value counters (4 digits upwards).	**Compare and order numbers:** Order these numbers in ascending order: 1890, 4850, 640, 2520 1. Copy the sequence carefully. Check. 2. Look at the number of digits and group them; for example, 3-digit numbers then 4-digit numbers. 3. Start in highest place value column, working across. Compare. 4. Rewrite in the correct order (ascending or descending). Cross out numbers as you go.
Conceptual variation		**Key vocabulary**
Children can apply their understanding by solving missing digit place value problems: Use the digit cards 1 to 5 to complete the comparisons. You can only use each digit card once. 1 2 3 4 5 564□ < □73□ 2□38 > 23□5 Put one digit in each box so that the list of numbers is in **ascending** order (smallest to greatest). Is there more than one solution? 1000s 100s 10s 1s 1 1 3 1 2 7 1 2 5 1 5 9 1 3 8 1 1 5	< > = Less than Greater than Equal to Children must be encouraged at all times to read number sentences aloud using the correct vocabulary. To do this successfully, children must first develop the understanding that number sentences are always read from left to right. 4 + 3 = 10 – 3 Four plus three is equal to ten take away three. 2 x 8 > 3 x 4 Two multiplied by eight is greater than 3 multiplied by 4. 3256 < 5116 Three thousand, two hundred and fifty-six is less than five thousand, one hundred and sixteen.	place value, number, digit, headings, place value chart, millions, hundred thousands, ten thousands, thousands, hundreds, tens, ones, tenths, hundredths, greater than, less than, equal to, of equal value to, what is the worth of… , what is the value of…, what is the place value of… , ascending, descending, largest, greatest, smallest, least, inequality statement, missing digit problem, more, less, count through zero, Roman numerals Where possible, please use Dienes apparatus and place value counters using **Singapore Maths colours**. This will ensure consistency across concrete and pictorial representations. Children should still be exposed to alternative representations and colours to develop fluency once new concepts have been taught. 

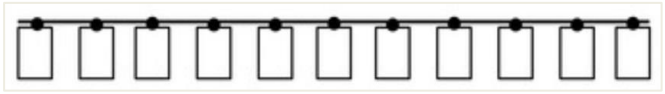
Add and subtract 1, 10, 100 and 1000 (place value)

Concrete	Pictorial	Abstract
Place value chart with no exchange: 3425 + 2000=  Use additional place value counters to add on 1s, 10s, 100s and 1000s without crossing boundaries:  This can also be replicated using Dienes. For children struggling to breach tens and hundreds, a number line or 100/200 square can be used to secure this knowledge:  	**No exchange:** 3425 + 2000=  If breaching tens, hundreds or thousands, refer to methods used in column addition as **regrouping** and **exchanging** will be required.	**No exchange:** 3425 + 2000=  Use a place value chart. Column headings until children are fluent. One digit per square. If breaching tens, hundreds or thousands, refer to methods used in column addition as **regrouping** and **exchanging** will be required.

Rounding to the nearest 10, 100 or 1000

Concrete

Number line:
27 rounded to the nearest 10.



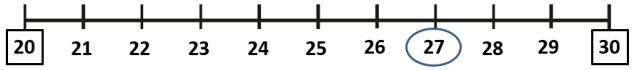
Use a physical number line (laminated) with a paperclip. This can be written on a wiped off using a drywipe marker as required. Chn can slide the paperclip left and right.

Which multiple of 10 is 27 nearest to?

"I can see that the multiple of 10 closest to 27 is 30."

Pictorial

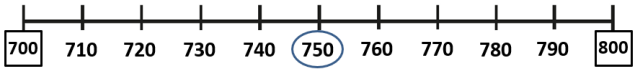
Number line:
27 rounded to the nearest 10.



1. Identify the two nearest multiples of 10
2. Circle the number you are rounding
3. Which multiple of 10 is it nearest to?
4. Write your answer using ≈

$$27 \approx 30$$

Round 750 to the nearest 100.



$$750 \approx 800$$

Chn to recognise that a number half-way between two multiples of 10, 100 and 1000 will always round up.

Abstract

Abstract method:
27 rounded to the nearest 10.

Step 2:

	H	T	O
		2	7

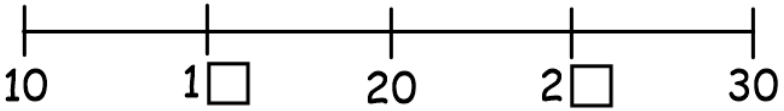
Step 3:

	H	T	O	
		2	7	≈
		3	0	

1. Write the number you are rounding using place value headings.
2. Use the rhyme:
   - Underline the digit
   - Go next door (arrow above decider)
   - 5 or higher, add one more
   - 4 or lower, just ignore
3. Rewrite the rounded digit underneath. Digits to the left stay the same. Digits to the right become placeholder zeros.
4. Write the ≈ sign

Conceptual variation

Problem solving:
Ron is rounding to the nearest 10. His answer is 20.
What is the smallest number that Ron could round to 20?
What is the largest number that Ron could round to 20?



Key vocabulary

place value headings, number, digit, underline, arrow, decider, placeholder zeroes, round up, round down, multiple, closest multiple, nearest multiple, largest/smallest number that rounds to, approximately

Roman numerals

Rules

1. ‘I’, ‘X’ or ‘C’ can be used together up to 3 times in a row.

I	X	C
1	10	100

III = 3
XXX = 30
CCC = 300

2. Count on with the numeral ‘I’ from V, X or L up to 3 times.

VIII = 8
XIII = 13
LIII = 53

3. When “left” is small and “right” is bigger, Subtract the “left” from “right-hand” figure.

4 is written 1 before 5.
9 is written 1 before 10.

V	X	L
5	10	50

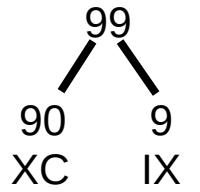
IV	IX	XL	XC
4	9	40	90

Steps to success

- Partition using place value.
- Look for any subtractions first:
  - If there is an I before a V or X, subtract 1.
  - If there is an X before L or C, subtract 10.

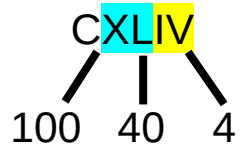
Abstract

Partition using place value:
Write 99 in Roman numerals.



99 = XCIX

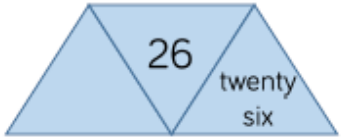
Write CXLIV in numerals.

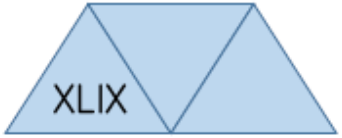


CXLIV = 144

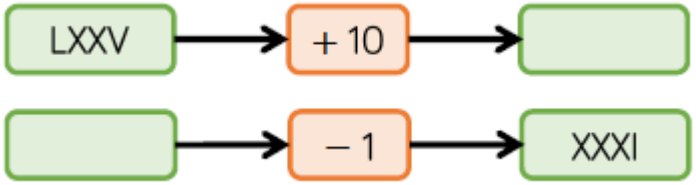
Conceptual variation

Fluency (numbers, numerals, words):





Function machines:



Reasoning:
Why is there no zero in Roman numerals? What might it look like?

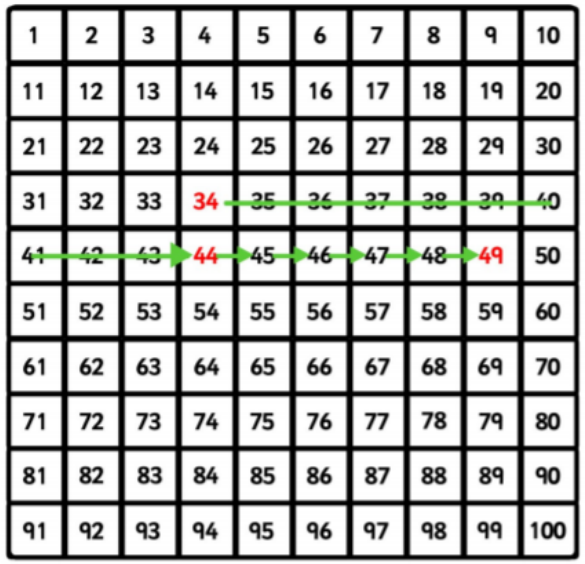
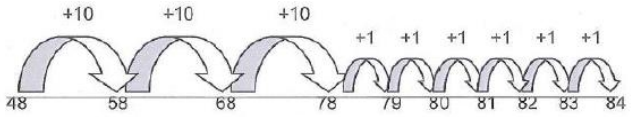
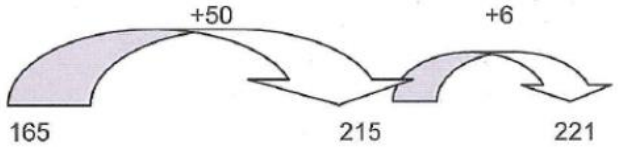
Can you spot any patterns? If 20 is XX, what might 200 be?

Key vocabulary

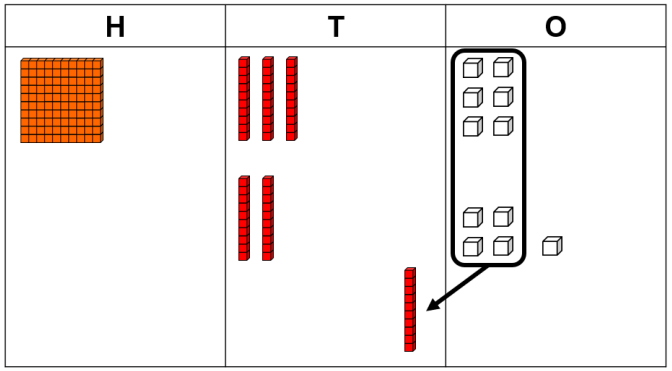
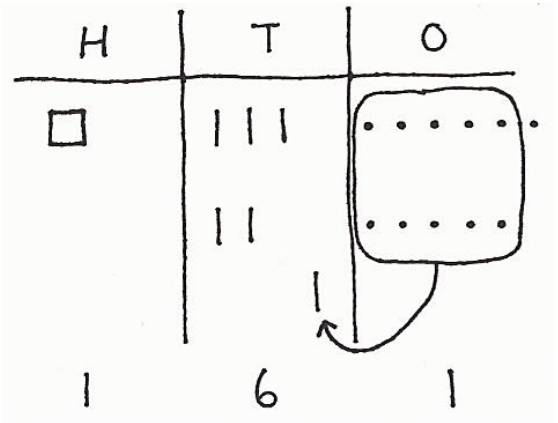
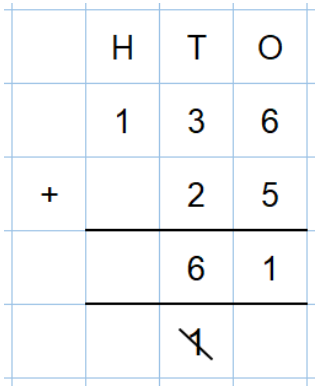
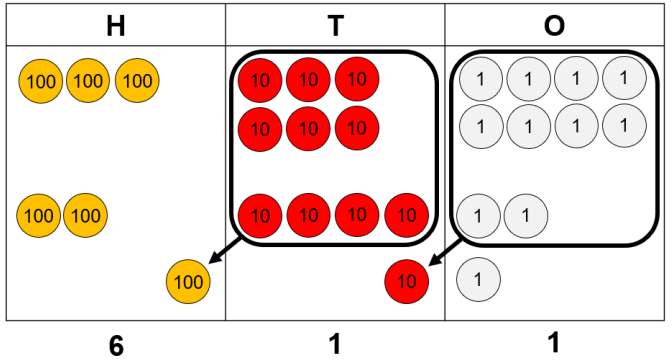
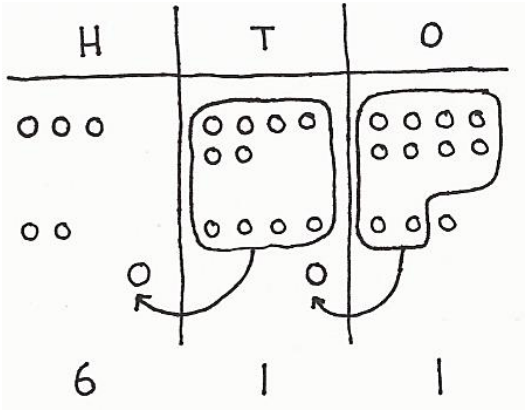
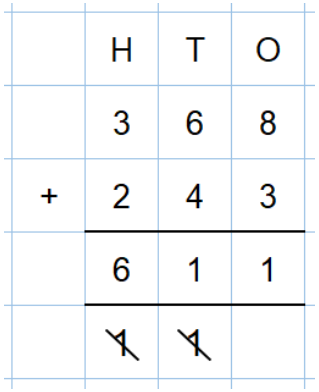
place value, partition, number, digit, numeral, subtractions, additions, count on

Calculation policy: addition

Mental methods

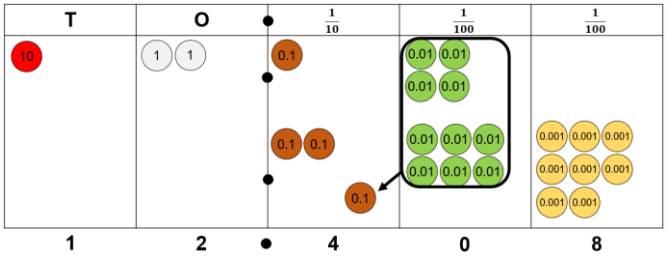
Concrete	Pictorial	Abstract
Number square with a counter/object: 34 + 15 =  	**Number line:** 48 + 36 = 1. Start with the largest number first (augend). 2. Partition the addend (30 + 6). 3. Count on (48 + 30 + 6).  165 + 56 =  	**Partition:** 48 + 36 = 1. Partition the augend and addend. 48 + 36 = 40 8 30 6 2. Add the tens. Add the ones. Add together. 40 + 30 = 70 8 + 6 = 14 70 + 14 = 84 **Partition one number:** 48 + 36 = 1. Partition the addend. 36 30 6 2. Add on the tens first. 48 + 30 = 78 3. Add on the ones. 78 + 6 = 84

Formal written method: column addition

Concrete	Pictorial	Abstract
Dienes (up to 3 digits): 136 + 25 =  When there are 10 ones in the O column, **regroup** then **exchange** for 1 ten. When there are 10 tens in the T column, **regroup** then **exchange** for 1 hundred.	**Dienes (up to 3 digits):** 136+25 =  Represent the Dienes apparatus pictorially e.g. lines for tens and dots for ones. Circle to **regroup** then complete the **exchange**. Show the **exchange** using an arrow.	**Formal written method:** 136 + 25 =  Operation presented on left hand side. Place value headings used until children are secure in place value. One digit per square. Work from right to left. Digits **regrouped** underneath. **Regrouped** digit to be crossed out diagonally from top left to bottom right.
Place value counters (4 digits upwards): 368 + 243 =  When there are 10 ones in the O column, **regroup** then **exchange** for 1 ten. When there are 10 tens in the T column, **regroup** then **exchange** for 1 hundred. Develop fluency and understanding by using blank counters.	**Place value counters (4 digit upwards):** 368 + 243 =  Children to represent the counters in a place value chart. Circle to **regroup** then complete the **exchange**. Show the **exchange** using an arrow.	**Formal written method:** 368 + 243 = 

Place value counters:

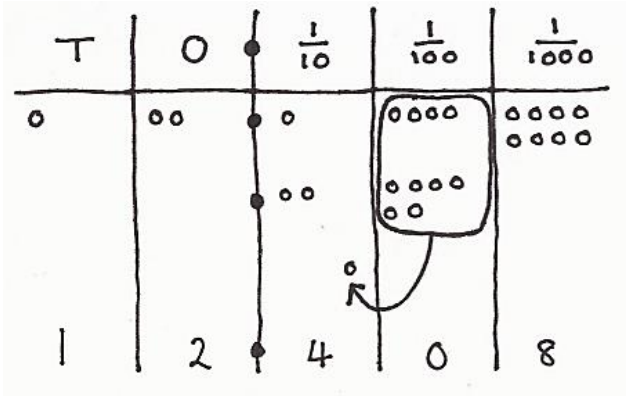
12.14 + 0.268 =



When there are 10 hundredths in the $\frac{1}{100}$ column, **regroup** then **exchange** for 1 tenth in the $\frac{1}{10}$ column. Develop fluency and understanding by using blank counters.

Place value counters (decimals):

12.14 + 0.268 =



Formal written method (decimals):

12.14 + 0.268 =

	T	O	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
	1	2	•	1	4	0
+		0	•	2	6	8
	1	2	•	4	0	8
				1		

One digit per square.

Decimal point on the line.

Place value headings represented as follows:

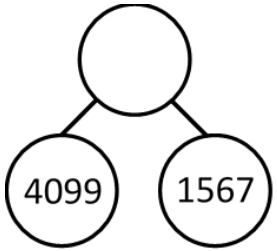
- $\frac{1}{10}$ – Tenths
- $\frac{1}{100}$ – Hundredths
- $\frac{1}{1000}$ – Thousandths

Children to align decimal point where the augend and addend have a different place value to each other.

Empty decimal places should be filled with a placeholder zero, where required, to show the place value in each column.

Conceptual variation

Part-whole model:



Bar model:

3110	707

Number sentence variations:

$\square = 21 + 34$

Children must understand that number sentences are always read from left to right. Calculations should be written either side of the equality sign so that the sign is not just interpreted as the answer.
"Something equals twenty-one plus thirty-four."

Worded number sentences:
Calculate the sum of twenty-one and thirty-four.

Balanced equations involving addition and subtraction:

$235 - 142 = \square + 50$

Word problems:
There are 172 non-fiction books in the school library and 356 fiction books. How many books are there in the library altogether?

Reasoning:

- 21 + 34 = 55. Prove it!
- What is the same/different? Is there a pattern? How will this affect your answer?
  - 346 + 100 =
  - 346 + 99 =
  - 346 + 101 =

Missing digit problems:

$$\begin{array}{r} 262 \\ +\ 3\square 1 \\ \hline 583 \end{array} \qquad \begin{array}{r} 322 \\ +\ 16\square \\ \hline 491 \end{array}$$

Use addition to calculate missing digit subtraction problems:

$\square - 24 = 59$

Add more than two numbers:
Break down into two separate calculations using column addition.
A + B + C = Total

	A
+	B
	Answer

	Answer
+	C
	Total

Key concepts and vocabulary

Concrete and pictorial representations must be used alongside the formal written method to ensure that the children are being taught the correct conceptual understanding.

At no time must the language "borrow" be used: we **exchange** and **regroup**.

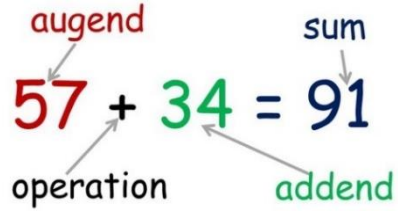
Pupils should use unitising language to describe within-column calculations:
"3 ones plus 5 ones is equal to 8 ones."
"4 tens plus 2 tens is equal to 6 tens."

Ensure that the children are aware that if you swap the augend and addend in addition, you will still calculate the same total (Commutative Law). It also does not matter how we group the numbers (Associative Law).

Encourage children to use calculating rather than counting.

add, addition, sum, total, plus, altogether, and, make, more, 'is equal to', 'is the same as', count on, regroup, regrouping, exchange, exchanging, boundary, calculate, calculation, column addition, formal method, place value, number, digit, operation, parts and wholes, bar model

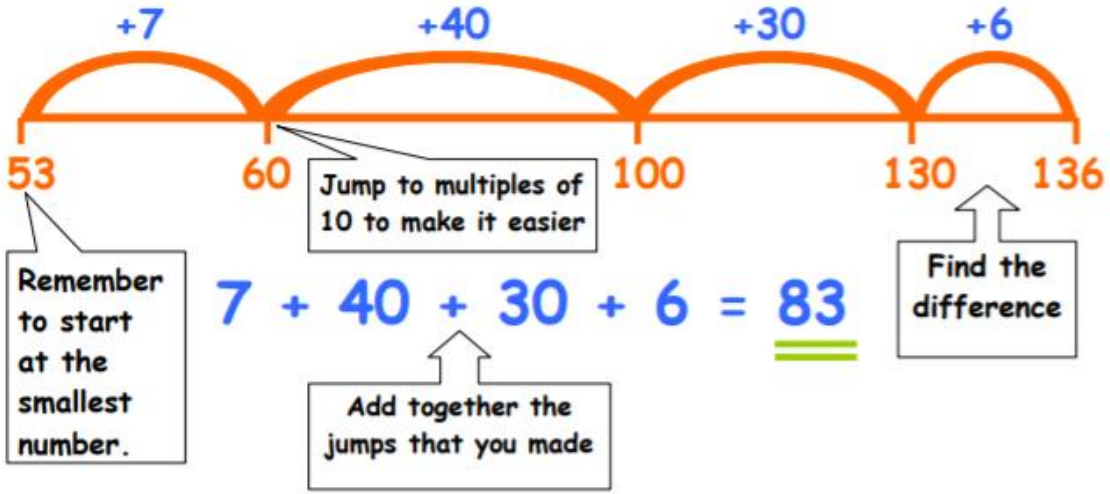
The following vocabulary is referred to throughout this policy but children and class teachers will <u>not</u> use this in teaching and learning:



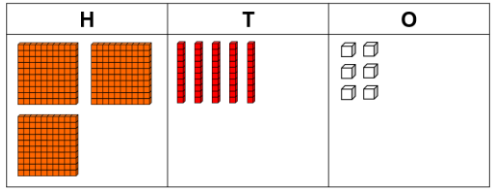
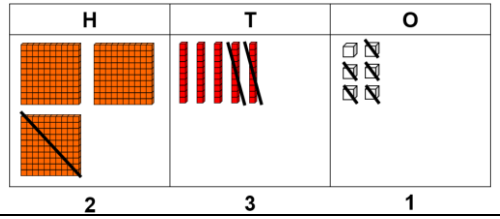
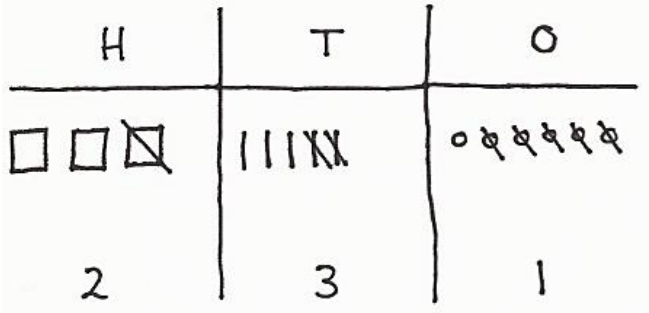
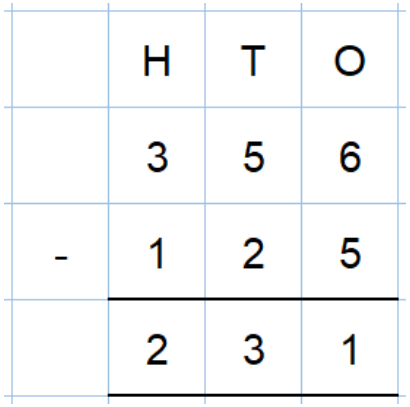
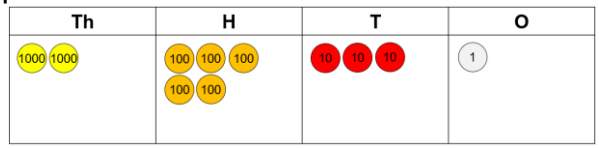
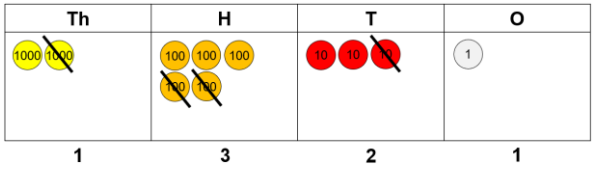
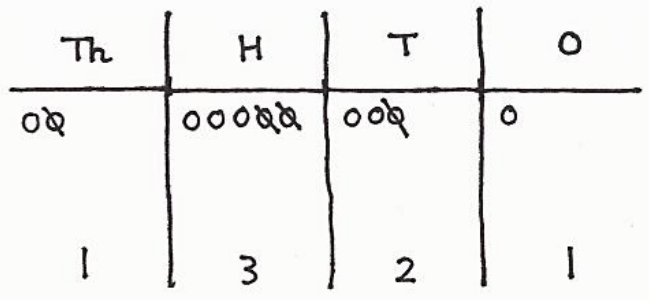
Sometimes both the augend and the addend are called addends. Sometimes the sum is called the total.

Calculation policy: subtraction

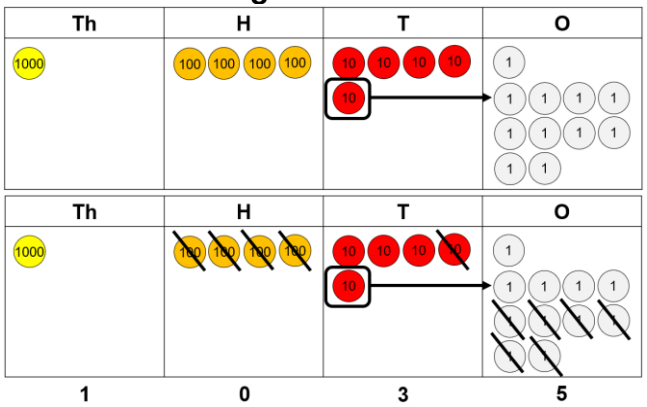
Mental methods

Concrete	Pictorial	Abstract
Number line (counting on): 136 – 53 =  -Always find 100 on the number line. Chn will be able to use their knowledge of number bonds to 100 to quickly count in multiples of 10.		**Count on mentally:** 136 – 53 = 1. Start with the subtrahend. 53 2. Count on to the next multiple of 10. 53 up to 60 (+7) 3. Count on in multiples of 10. Stop at the multiple of 10 before the minuend. (Children can jump to the nearest multiple of 100 and use number bonds to support.) 60 up to 130 (+70) 4. Count on to the minuend. 130 (+6) 5. Total up to find the difference: 7 + 70 + 6 = 83

Formal written method: column subtraction

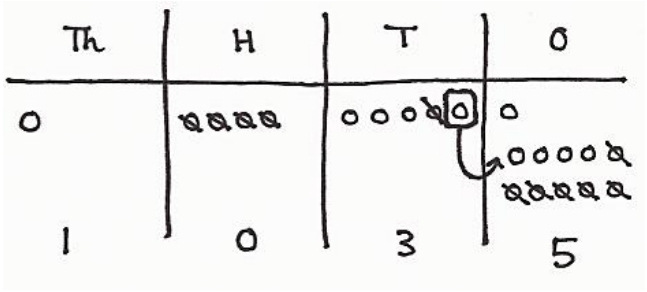
Concrete	Pictorial	Abstract
Dienes (up to 3 digits): 356 – 125 = 1. Represent the minuend using Dienes on a place value chart.  2. Subtract from right to left. Dienes to be physically removed from the place value chart to show the subtraction. Difference written underneath.  	**No exchange (three digits):** 356 – 125 =  Represent the Dienes apparatus pictorially e.g. lines for tens and dots for ones. Cross out to show the subtraction.	**No exchange (three digits):** 356 – 125 =  Place value headings used (until children are secure). One digit per square. Operation on the left-hand side. Work from right to left. Top digit subtract the bottom digit.
Place value counters (4 digits upwards): 2531 – 1210 = 1. Represent the minuend using counters on a place value chart.  2. Subtract from right to left. Counters to be physically removed from the place value chart to show the subtraction. Difference written underneath.  	**No exchange (three digits):** 2531 – 1210 =  Children to represent the counters in a place value chart. Cross out to show the subtraction.	**No exchange (three digits):** 2531 – 1210 = Place value headings used (until children are secure). One digit per square. Operation on the left-hand side. Work from right to left. Top digit subtract the bottom digit.

Place value counters (4 digits upwards):
1451 – 416 =
Show the **exchange**:



Counters to be physically removed from the place value chart to show the subtraction.

No exchange (three digits):
1451 – 416 =



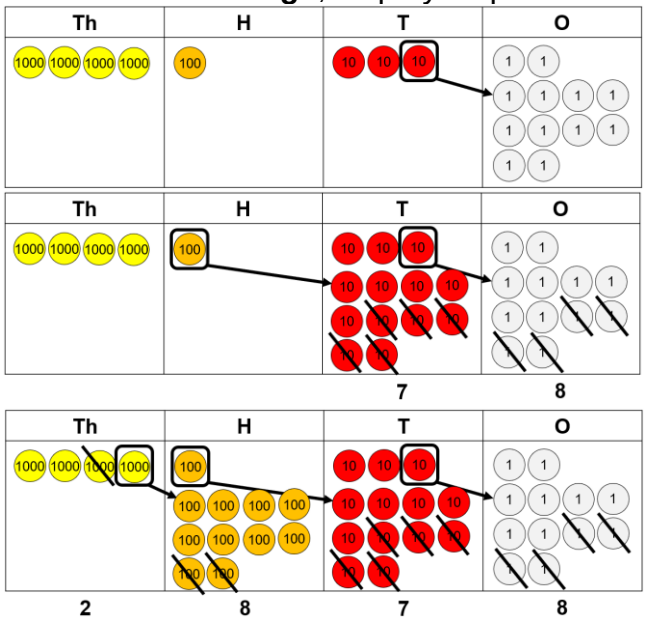
Circle to complete the **exchange**. Show the **exchange** using an arrow. Cross out to show the subtraction.

No exchange (three digits):
1451 – 416 =

	Th	H	T	O
	1	4	4 5	1 1
-		4	1	6
	1	0	3	5

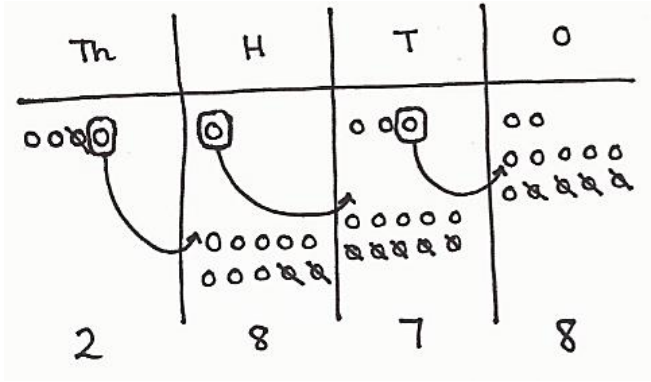
Exchanged digit to be crossed out diagonally from top left to bottom right. Regrouped digits written top left-hand side of the square.

Place value counters (4 digits upwards):
4132 – 1254 =
Show each **exchange**, step by step:



Counters to be physically removed from the place value chart to show the subtraction.

More than one exchange (neighbouring columns):
4132 – 1254 =



Circle to complete the **exchange**. Show the **exchange** using an arrow. Cross out to show the subtraction.

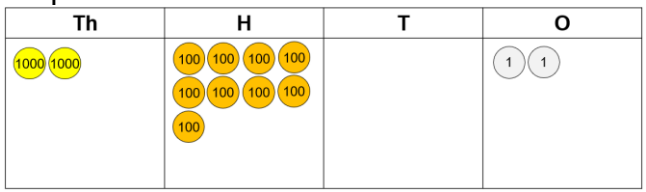
More than one exchange (neighbouring columns):
4132 – 1254 =

	Th	H	T	O
	3 4	10 1	12 3	1 2
-	1	2	5	4
	2	8	7	8

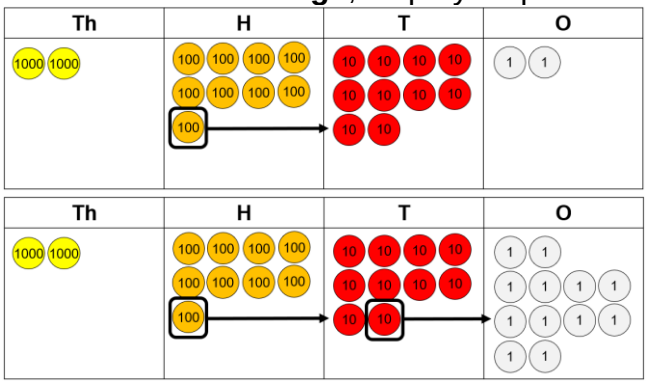
Exchanged digit to be crossed out diagonally from top left to bottom right. Regrouped digits written top left-hand side of the square.

Place value counters (4 digits upwards):
2902 – 1368 =

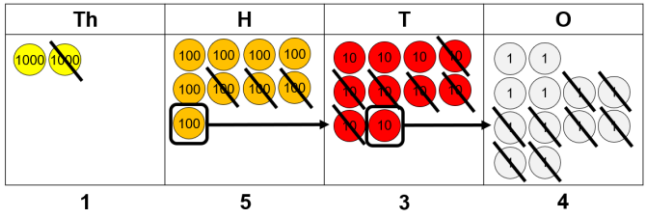
1. Represent the minuend using counters on the place value chart.



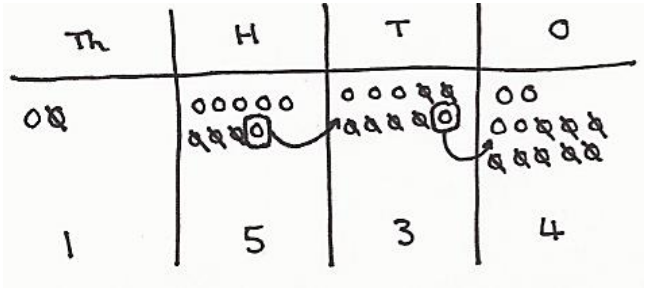
2. Show each **exchange**, step by step:



3. Complete the subtraction. Counters to be physically removed from the place value chart to show the subtraction.



Subtract across zero:
2902 – 1368 =



Circle to complete the **exchange**. Show the **exchange** using an arrow. Cross out to show the subtraction.

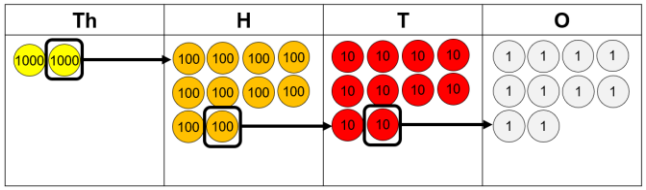
Subtract across zero:
2902 – 1368 =

	Th	H	T	O
	2	8 ~~9~~	9 ~~0~~	1 2
-	1	3	6	8
	1	5	3	4

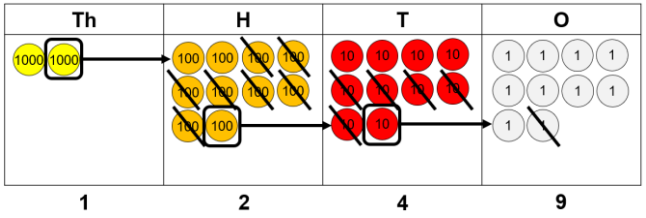
When there is **more than one exchange**, ensure that both digits are crossed through from top left to bottom right.

Place value counters (4 digits upwards):
2000 – 751 =

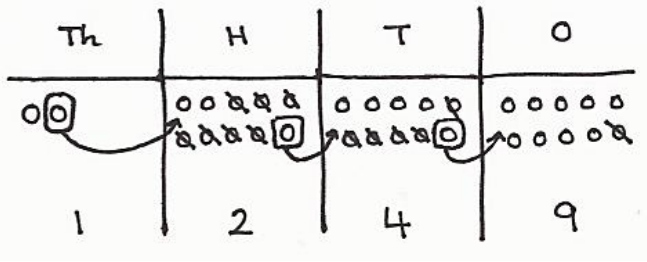
Show each **exchange**, step by step:



Then complete the subtraction. Counters to be physically removed from the place value chart to show the subtraction.



Subtract across zero (multiple of 100 or 1000):
2000 – 751 =



Circle to complete the **exchange**. Show the **exchange** using an arrow. Cross out to show the subtraction.

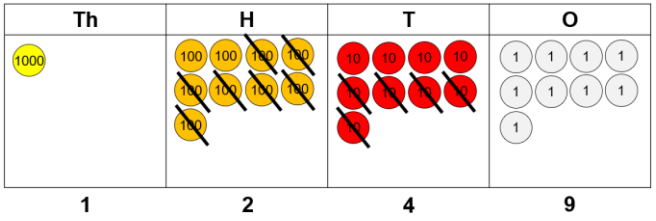
Subtract across zero (multiple of 100 or 1000):
2000 – 751 =

	Th	H	T	O
	1	9	9	10
-		7	5	1
	1	2	4	9

Work methodically. Regroup from Th to H, H to T, T to O in the correct order. Children must be aware that you can only **exchange** with the column <u>next door</u> as modelled using concrete / <u>pictorial</u> representations.

Use compensation method to subtract with an exchange of more than one zero:
2000 – 751 =

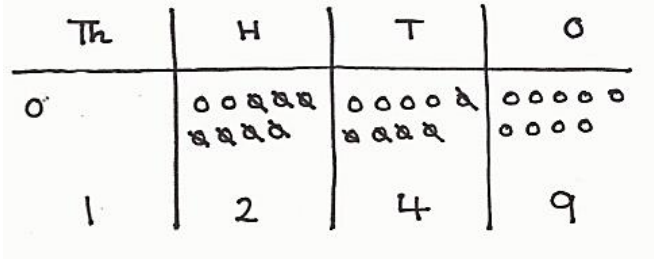
Subtract 1 from both the minuend and subtrahend: 1999 – 750.



No need for any exchange using the compensation method.

Use compensation method to subtract with an exchange of more than one zero:
2000 – 751 =

Subtract 1 from both the minuend and subtrahend: 1999 – 750.



No need for any exchange using the compensation method.

Use compensation method to subtract with an exchange of more than one zero:
2000 – 751 =

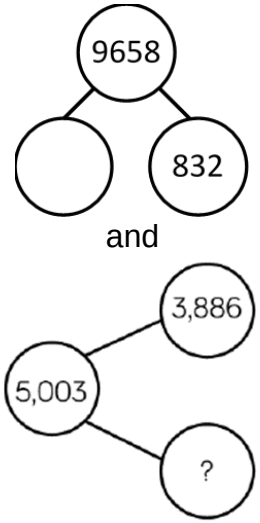
Subtract 1 from both the minuend and subtrahend: 1999 – 750.

	Th	H	T	O
	1	9	9	9
-		7	5	0
	1	2	4	9

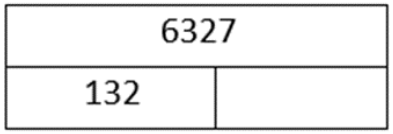
For children exceeding year group expectations only: use once children have understood how to subtract across zero with multiples of 1000 and 100 conceptually.

Conceptual variation

Part-whole model:



Bar model:



Number sentence variations:

$\square = 438 - 20$

Children must understand that number sentences are always read from left to right. Calculations should be written either side of the equality sign so that the sign is not just interpreted as the answer. “Something equals four hundred and thirty-eight take away twenty.”

Word problems:

Dora and Mo are collecting book tokens. Dora has collected 1452 tokens. Mo has collected 621 fewer tokens than Dora. How many tokens does Mo have?

Reasoning:

Aisha is calculating 7585 – 316. Do you agree with her? Explain your answer.

	Th	H	T	O
	7	5	8	5
–	3	1	6	
	4	4	2	5

What is the same/different? Is there a pattern? How will this affect your answer?

- 907 – 100 = 807
- 907 – 99 = ___
- 907 – 101 = ___

Missing digit problems:

	TH	H	T	O
	7	3	?	4
-	5	1	5	9
	?	2	3	5

Key concepts and vocabulary

Concrete and pictorial representations must be used alongside the formal written method to ensure that the children are being taught the correct conceptual understanding.

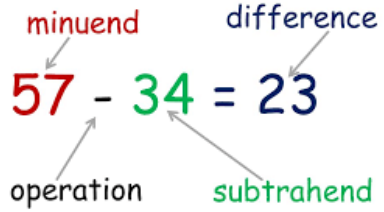
Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.

At no time must the language “borrow” be used: we **exchange**.

When learning to **exchange**, explore partitioning in different ways so that children understand that when we exchange, the value is the same; for example, 72 is 70 and 2 but it is also 60 and 12 or 50 and 22. Emphasise that the value does not change; we have just partitioned it in a different way.

subtract, subtraction, take, take away, less, minus, difference, leaves, distance between, how many more, how many less, how many fewer, most, least, count back, count forward, how many left, how much less is…, exchange, exchanging, boundary, calculate, calculation, column subtraction, formal method, place value, number, digit, operation, parts and wholes, bar model

The following vocabulary is referred to throughout this policy but children and class teachers will <u>not</u> use this in teaching and learning:



Worded number sentences:
Find the difference between…
Calculate the difference between…
Subtract ___ from ___ .

Balanced equations involving addition and subtraction:

235 – 142 = ☐ + 50

Subtract more than two numbers:
8359 – 925 – 192 =

8359		
925	192	?

Break down into two separate calculations using column subtraction.

Use subtraction to calculate missing digit addition problems:

☐ + 400 = 826

50 + ☐ = 76

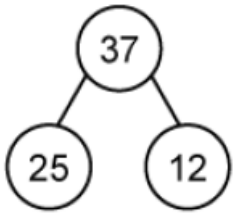
Developing fluency using the inverse operation

Children's understanding should go beyond the fact that addition and subtraction are inverse operations. They need understand how the terms in addition and subtraction equations are related to each other, and to the parts and whole within an additive relationship and use this understanding to manipulate equations.

sum	
addend	addend

minuend	
subtrahend	difference

minuend	
difference	subtrahend



37	
25	12

Figure 24: partitioning diagrams showing the additive relationship between 25, 12 and 37

$25 + 12 = 37$
$12 + 25 = 37$
$37 = 25 + 12$
$37 = 12 + 25$

$37 - 12 = 25$
$37 - 25 = 12$
$25 = 37 - 12$
$12 = 37 - 25$

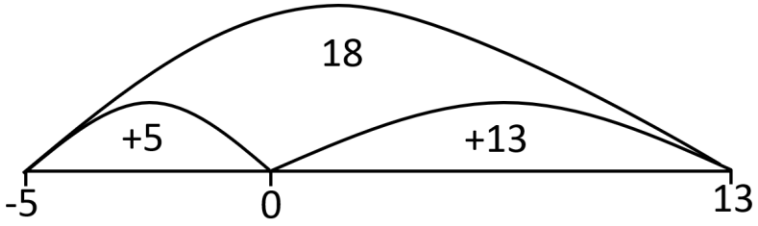
"If we swap the values of the subtrahend and difference, the minuend remains the same."

Add and subtract negative numbers

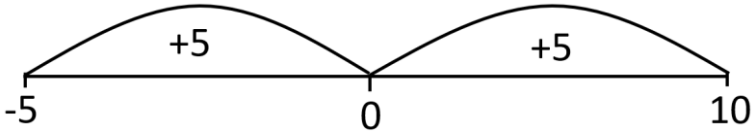
Pictorial

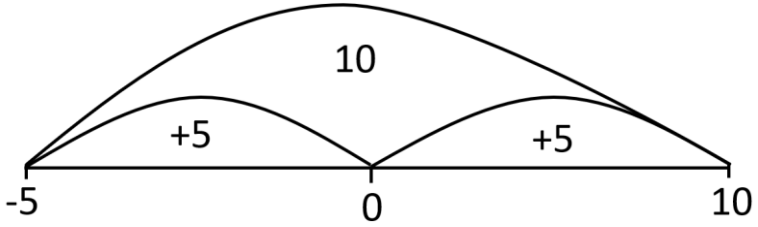
Always use pictorial method when adding and subtracting negative numbers:

-5 + 18 (negative five plus eighteen) =



10 – 15 =



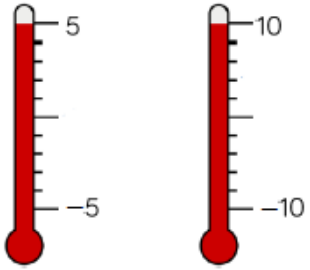


- Count up from negative to positive.
- Always fill in zero on the number line.

Conceptual variation

Developing fluency:

Vertical number line (thermometer, car park, ocean depth etc.)



Worded problems:

The temperature in Svalbard is -53°c. The temperature in Abisko is -17°c. Find the difference between these temperatures.

Diane's bank account had gone £156 overdrawn. She paid in £200 cash into her account and was charged a £25 fee. What was her new bank balance?

Problem solving:

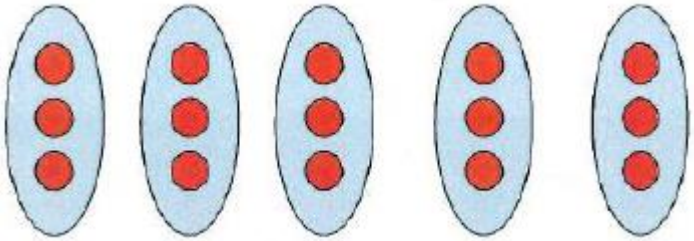
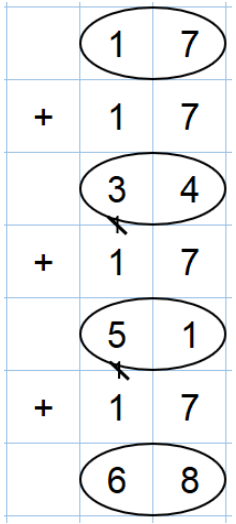
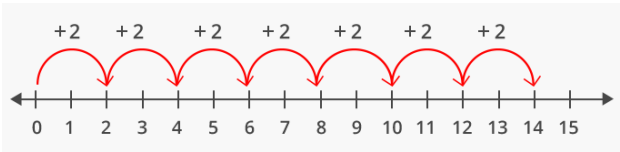
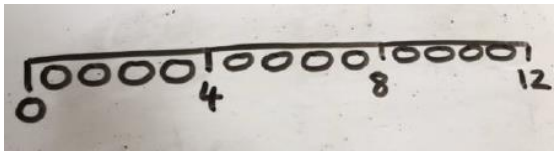
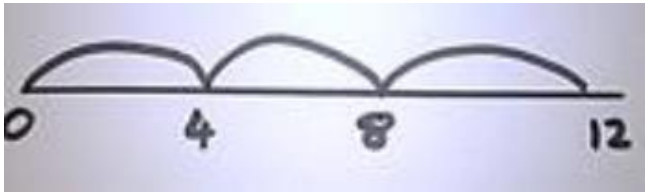
Dai works in an underground carpark. All of the floors are 10 metres apart. Dai's office is on floor -4. He takes a lift up to -3 to fix a broken light bulb. He then goes down in the lift to floor -12 to repair a ticket machine. Finally, he takes the lift back to his office. In metres, how far has Dai travelled in the lift?

Key vocabulary

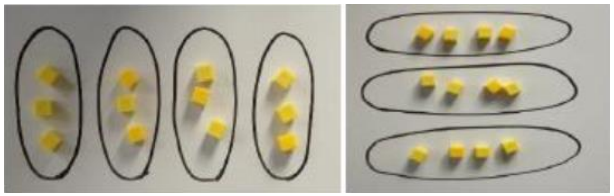
Children to read -5 as **negative five**, not minus five. This is to avoid confusion with the vocabulary linked to subtraction; for example, seven minus five equals two.

Calculation policy: multiplication

Mental methods and number facts

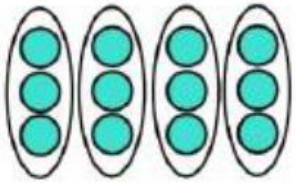
Concrete	Pictorial	Abstract
Skip counting with apparatus (repeated addition): Children use fingers to skip count to develop an understanding of grouping. Multi-link cubes, bead strings and number lines can also be used to skip count using grouping: 	**Skip counting using arrays (repeated addition):** Skip counting in threes. 3 + 3 + 3 + 3 + 3… 	**Skip counting (multiplication facts):** Skip count horizontally. Use multiplication facts. 2 4 6 8 10 12… **Skip counting (beyond 12):** Skip count vertically. Use repeated addition. Circle every multiple. 
Number line to show repeated groups: Counting in twos. 	**Number line represented pictorially to show repeated groups:** Counting in fours. 	**Abstract number line showing three jumps of four:** 3 x 4 = 12 

Multiplication facts as arrays:
Multiplication shown as commutative:
3 x 4 = 12
4 x 3 = 12

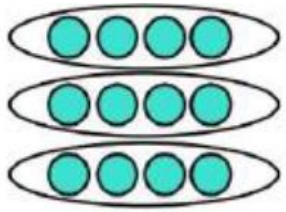


3 x 4 = 4 x 3

Multiplication facts as arrays:
Multiplication shown as commutative:
3 x 4 = 12
4 x 3 = 12

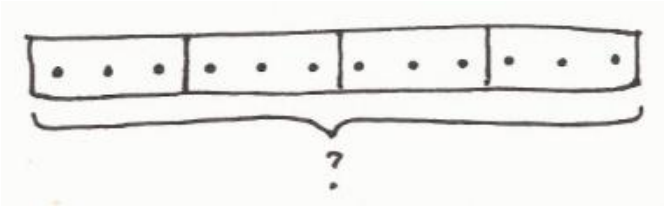


4 groups of 3
4 x 3 = 12



3 groups of 4
3 x 4 = 12

Children to represent concrete resources in a picture and use a bar model:



Multiplication facts:
Children have instant rapid recall of multiplication facts from memory, including an understanding of the commutative law:

3 x 4 = 12
4 x 3 = 12

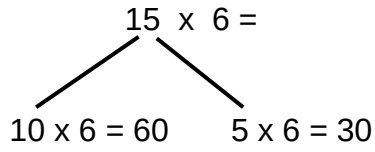
Make connection to repeated addition:

3 + 3 + 3 + 3 = 12

Children to be able to use an array to write a range of different calculations:
3 x 4 = 12
12 = 4 x 3
3 + 3 + 3 + 3 = 12
12 = 4 + 4 + 4

Distributive law for mental multiplication:
15 x 6 =

1. Partition the multiplicand then multiply.

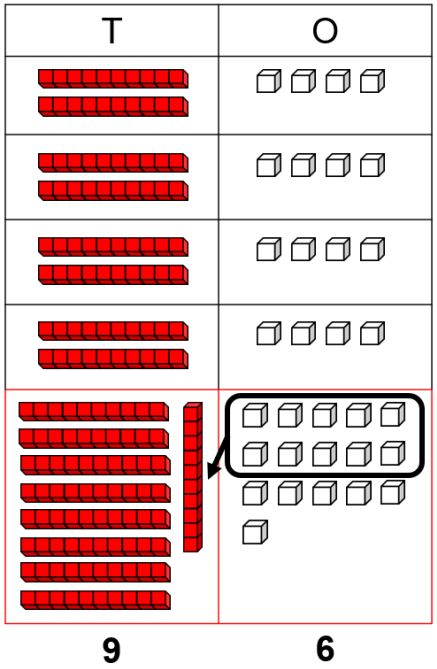
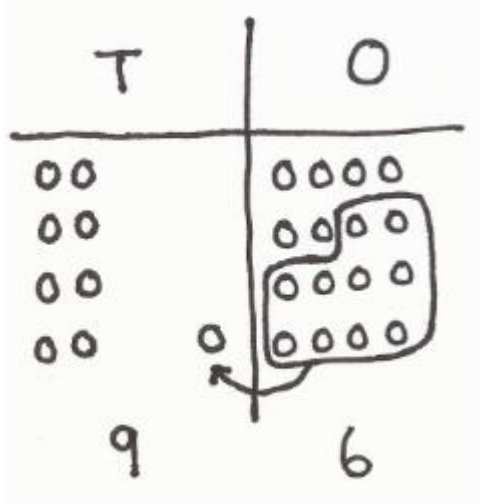
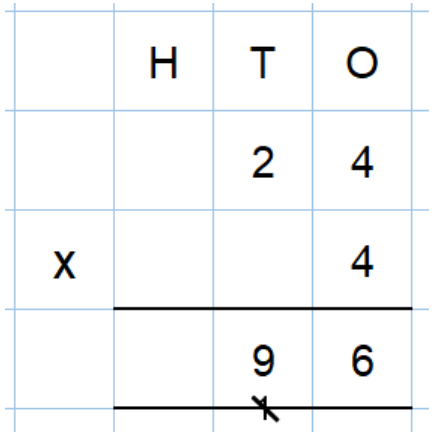


2. Add the product of each calculation together.

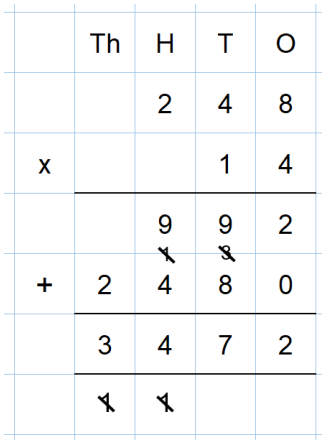
60 + 30 = 90

The distributive law dictates that
15 x 6 = 10 x 6 + 5 x 6.

Formal written method: short multiplication

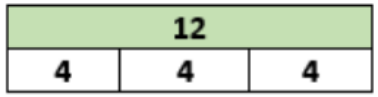
Concrete	Pictorial	Abstract
2 digits multiplied by a single digit (column method): 24 x 4 = Represented using concrete apparatus using Dienes:  	**2 digits multiplied by a single digit (column method):** Children represent counters pictorially:  **Alternatively, use the grid method:** Use the grid method, detailed above, as a pictorial progression to detail the individual calculations taking place using the column method of short multiplication. Children could be asked to work out a given calculation using the grid and then compare it to the teacher's column method. Discuss what the similarities and differences are. Go through the steps and use as success criteria.	**2 digits multiplied by a single digit (column method):** 24 x 4 =  Operation presented on left hand side. Place value headings used until children are secure in place value. One digit per square. Work from right to left. Digits **regrouped** with a small digit on the line – do not skip a line. **Regrouped** digit to be crossed out diagonally from top left to bottom right once used in the calculation.

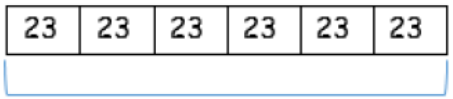
Formal written method: long multiplication

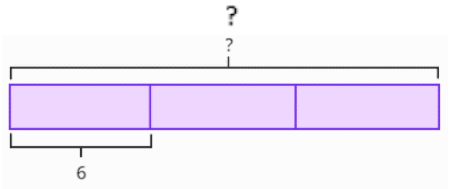
Concrete	Pictorial	Abstract
See above.	See above.	**Multiply by 2 digits (column method):** 248 x 14 =  Operation presented on left hand side. Place value headings used until children are secure in place value. One digit per square. Work from right to left. Digits **regrouped** with a small digit on the line – do not skip a line. **Regrouped** digit to be crossed out diagonally from top left to bottom right once used in the calculation. It is essential that children do not forget to include the placeholder zero when multiplying by the tens digit of the multiplier.
		If multiplying decimals: Remove the decimal point and place back in at the end of the calculation. 0.5 x 28 = 1. Made a note of the number of decimal places (1dp). 2. Complete the multiplication without the decimal: 5 x 28 = 140. 3. Adjust by 1 decimal place at the end (divide by 10): 0.5 x 28 = 14

Conceptual variation

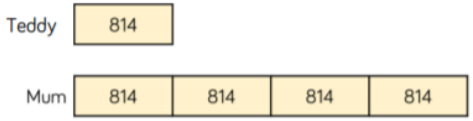
Bar model variations:



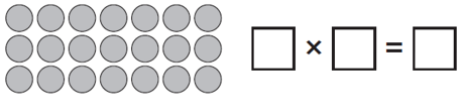




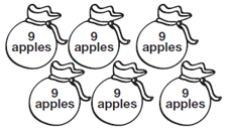
Teddy's mum um read 4 times as many pages as Teddy. How many pages did they read altogether? How many fewer pages did Teddy read?



Pictorial representations:



□ × □ = □



□ × □ = □

Worded number sentences:

Find the product of 6 and 23.
Multiply 23 and 6.
What is the product of 6 and 23?

Worded problems:

May had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Reasoning:

With counters, prove that 6 x 23 = 138.

What is the calculation?
What is the product?

Missing number problems:

$4 \times \square = 32$

Use multiplication to solve missing digit division problems (using the inverse):

$\square \div 3 = 36$

$619 = \square \div 8$

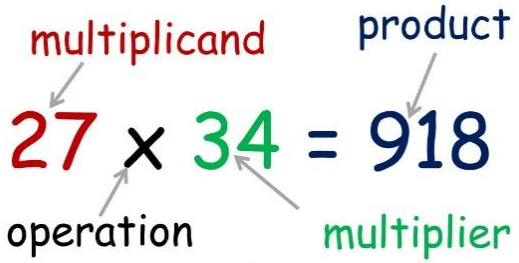
Key vocabulary

Based on their existing additive knowledge, children should be able to represent equal-group contexts with repeated addition expressions, for example, 5 + 5 + 5. They should then learn to write multiplication expressions to represent the same contexts, for example 3 x 5. Children must be able to explain how each term in a multiplication expression links to the context it represents. Children must also be able to understand equivalence between a repeated addition expression and a multiplication expression: 5 + 5 + 5 = 3 x 5.

Children should then learn to calculate the total number of items (the product), for contexts based on the multiplication tables, initially by skip counting. They should be able to write complete multiplication equations, for example 3 x 5 = 15, and explain how each term links to the context.

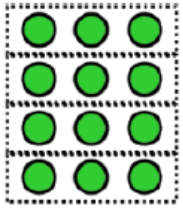
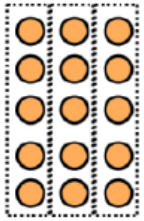
As children develop automatic recall of multiplication facts, they should learn how to use these to derive division facts.

Children should learn and use the correct names for the terms in multiplication equations:



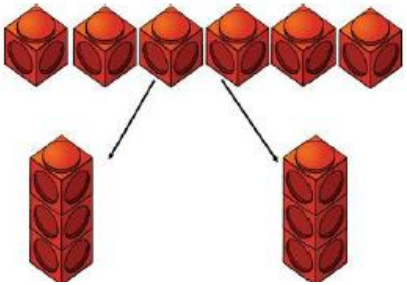
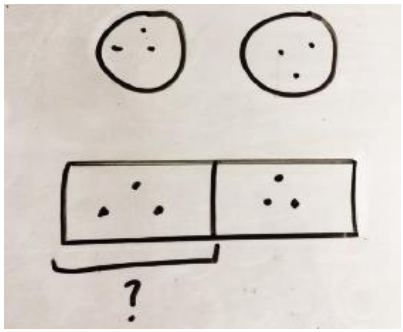
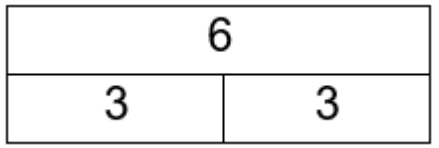
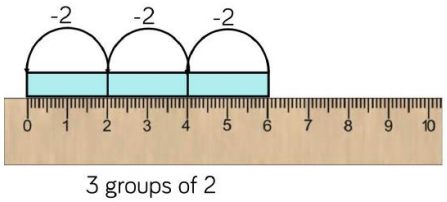
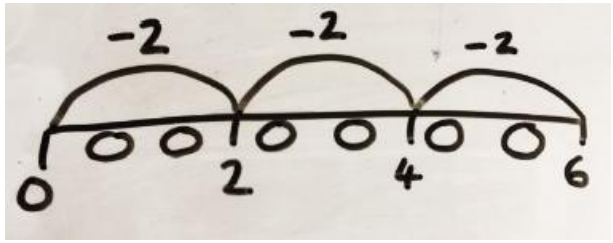
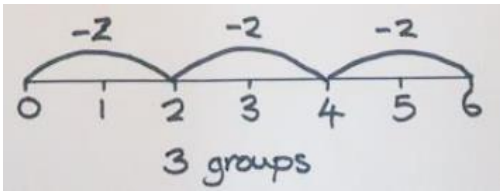
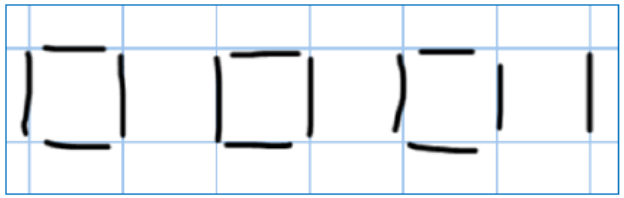
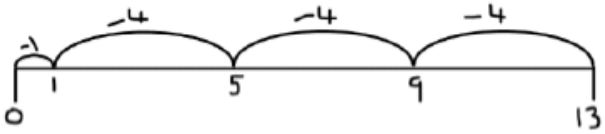
Both the multiplicand and multiplier are also factors.

multiply, multiplication, times, multiplication table, times table, product, groups of, lots of, altogether, multiply, scale up, skip count, double, twice, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, --- times as big as, once, twice, three times …. partition, grid method, array, short multiplication, long multiplication, exchange, regroup, place value, placeholder zero, digit, number, operation, multiple, multiples, factor, factors, common multiples, common factors, square, cube, decimal, prime numbers, prime factors, square number, cubed number, inverse, integer

Number sentence variations: $\square = 6 \times 23$ Children must understand that number sentences are always read from left to right. Calculations should be written either side of the equality sign so that the sign is not just interpreted as the answer. "Something equals six multiplied by twenty-three."	**Balanced equations:** $3 \times 4 = 4 + 4 + 4$ $3 \div \square = \square \times 3$ **Problems involving arrays:** There are 4 bags of sweets with 3 sweets in each bag. How many sweets are there altogether?  There are 3 school bags with 5 books in each one. How many books are there altogether? 	They must be able to use these 'known division facts' to solve division calculations, instead of using the skip counting method: "4 times 5 is 20, so 20 divided by 5 is 4." Ensure that the children are aware that if you swap the multiplicand and the multiplier in multiplication, you will still calculate the same product (Commutative Law).	

Calculation policy: division

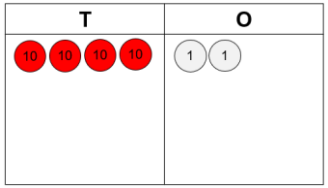
Mental methods and number facts

Concrete	Pictorial	Abstract
Sharing using a range of objects: 6 ÷ 2 = 	**Represent the sharing pictorially:** 6 ÷ 2 = 	**Number facts:** 6 ÷ 2 =  Children should also be encouraged to use their 2 times tables facts
Repeated subtraction using a ruler: 6 ÷ 2 =  3 groups of 2 Cuisenaire rods can also be used above the ruler.	**Represent repeated subtraction pictorially:** 6 ÷ 2 = 	**Abstract number line to represent the equal groups that have been subtracted:** 6 ÷ 2 = 
Grouping using a range of objects: 13 ÷ 4 = Use of lollipop sticks to form wholes – squares are made because we are dividing by 4. There are 3 whole squares with 1 stick left over.	**Children to represent lollipop sticks pictorially:**  There are 3 whole squares with 1 stick left over.	**Children to use multiplication facts:** 13 ÷ 4 = 3 r 1 Children should be encouraged to use their times tables facts; they could also represent repeated subtraction on a number line.  3 groups of 4 with 1 left over.

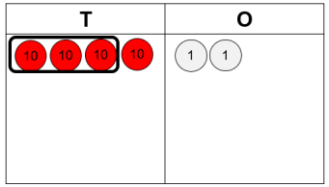
Grouping using place value counters:

42 ÷ 3 = 14

1. Represent the dividend in a place value chart.

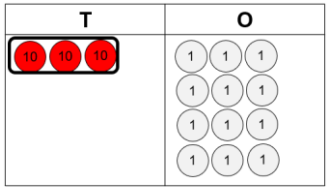


2. How many groups of 3 can be made in the tens column? There is one group of 3 in 4 tens.



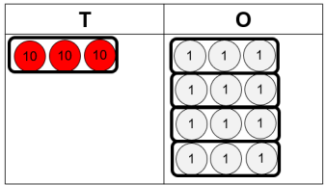
1

3. How many tens left over? 1 ten. Exchange 1 ten for ten ones.



1

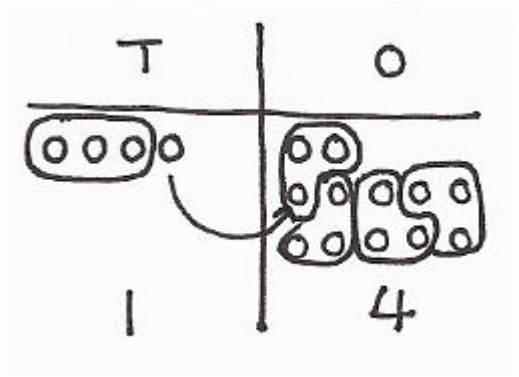
4. How many groups of 3 can be made in the ones column? There are 4 groups of 3 in 12 ones.



1 4

Children to represent the place value counters pictorially:

42 ÷ 3 =



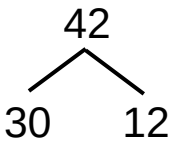
Use the same steps to success as the concrete method.

It is important that children understand we work from left to right in terms of place value for division, which is the opposite of addition, subtraction and multiplication.

Children to use the pictorial representation to make sense of the place value counters and write calculations to show the process:

42 ÷ 3 =

1. 42 ÷ 3
2. Partition the dividend into multiples of the divisor.



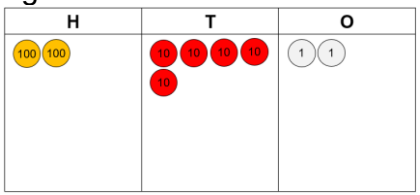
3. 30 ÷ 3 = 10
   12 ÷ 3 = 4
4. 10 + 4 = 14

Formal written method: short division

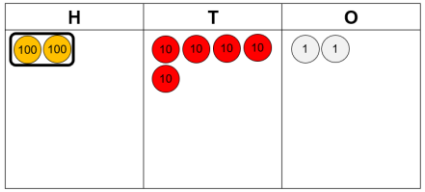
Concrete

Place value counters (no remainder):
252 ÷ 2 =

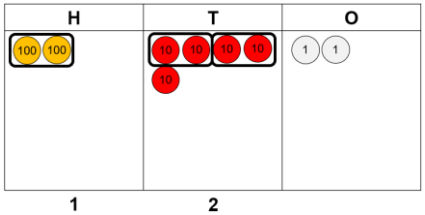
1. Represent the dividend in a place value chart using counters:



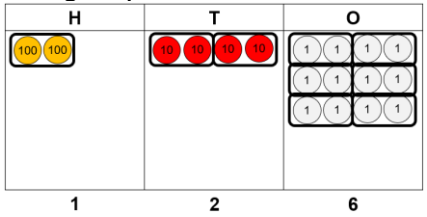
2. How many groups of 2 can be made in the hundreds column? There is 1 group of 2 hundreds.



3. How many groups of 2 can be made in the tens column? There are 2 groups of 2 tens. There is 1 ten leftover.

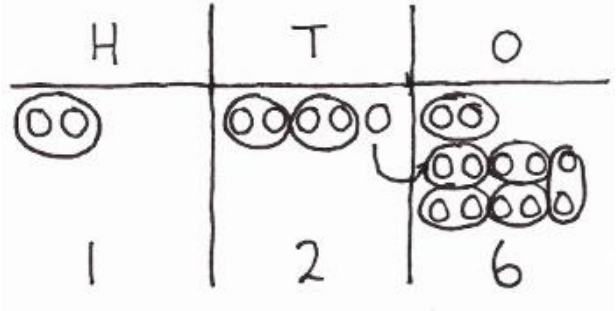


4. Exchange 1 ten for ten ones. How many groups of 2 can be made in the ones column? There are 6 groups of 2 ones.



Pictorial

Place value counters (no remainder):
252 ÷ 2 =



Use the same steps to success as the concrete method. **Exchange** shown using an arrow.

It is important that children understand we work from left to right in terms of place value for division, which is the opposite of addition, subtraction and multiplication.

Abstract

Bus stop (no remainder):
252 ÷ 2 =

	1	2	6
2	2	5	$^{1}2$

Dividend inside bus stop.
Quotient outside bus stop.
One digit per square.
Regroup top left-hand side of the square.

Use the language of grouping:
How many groups of 2 in 2?
How many groups of 2 in 5?

Bus stop (no remainder):
210 ÷ 5 =

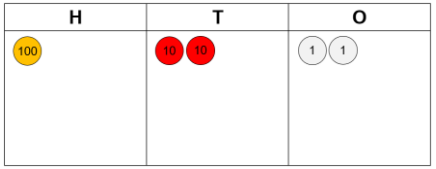
	0	4	2
5	2	$^{2}1$	$^{1}0$

Dividend inside bus stop.
Quotient outside bus stop.
One digit per square.
Regroup top left-hand side of the square.

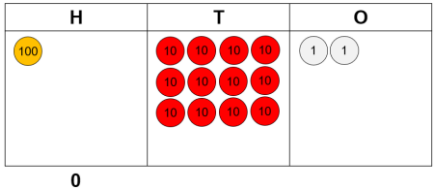
Use the language of grouping:
How many groups of 5 in 2?
How many groups of 5 in 21?

Place value counters (remainder):
122 ÷ 4 =

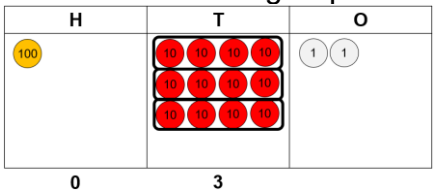
1. Represent the dividend in a place value chart using counters:



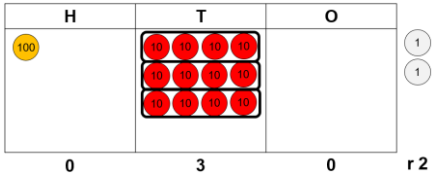
2. How many groups of 4 can be made in the hundreds column? There are no groups of 4 hundreds. Regroup 1 hundred in the tens column:



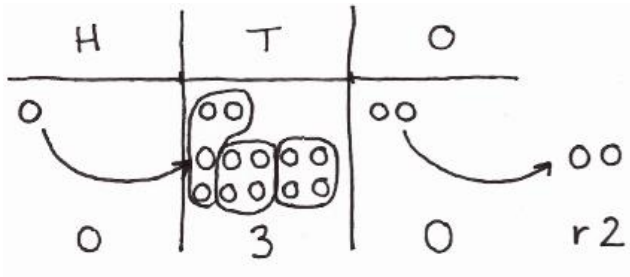
3. How many groups of 4 can be made in the tens column? There are 3 groups of tens.



4. How many groups of 4 can be made in the ones column? There are no groups of 4 ones. This leaves 2 ones as a remainder.



Place value counters (remainder):
122 ÷ 4 =



Use the same steps to success as the concrete method. **Exchange** shown using an arrow.

It is important that children understand we work from left to right in terms of place value for division, which is the opposite of addition, subtraction and multiplication.

Bus stop (remainder):
122 ÷ 4 =

	0	3	0	r 2
4	1	$^{1}2$	2	

Short division with remainders unless otherwise stated:

- Answer written as a fraction:

$30\frac{2}{4}$ or $30\frac{1}{2}$

- Answer as a decimal (to 2dp maximum):

	0	3	0 • 5
4	1	$^{1}2$	2 • $^{2}0$

If required, children to skip count multiplication facts horizontally. Use an arrow to signify where children are counting up to:

18 ÷ 4 = 4 r 2

4 8 12 16 20…

18

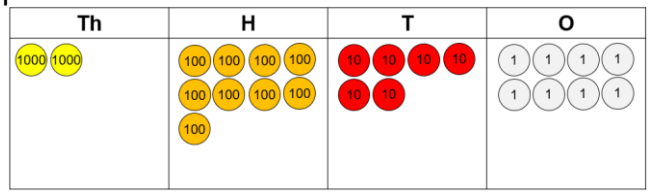
Formal written method: long division

Concrete

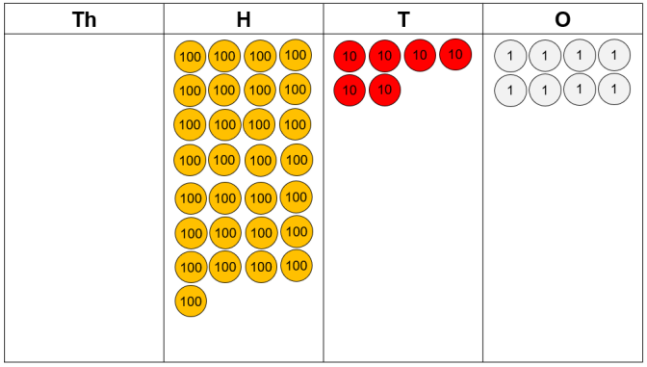
Long division using place value counters:

2968 ÷ 14 =

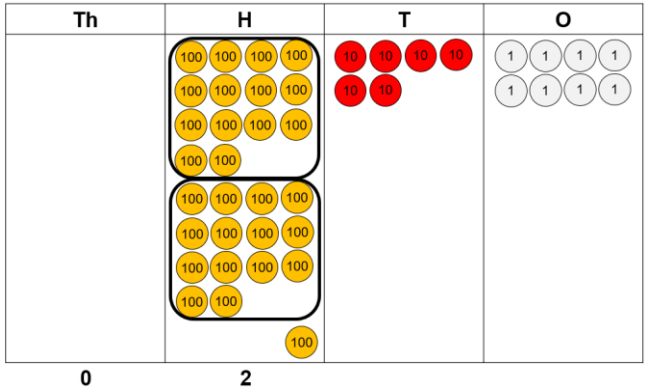
1. Represent the dividend using counters in a place value chart:



2. We cannot group 2 thousands in groups of 14 so we need to exchange them:

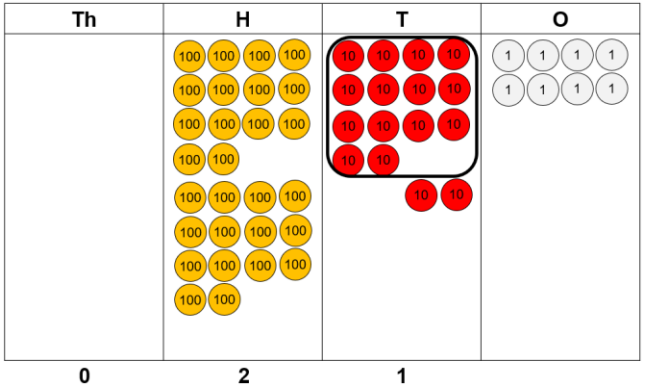


3. There are 2 groups of 14 in the hundreds column. There is 1 hundred left.

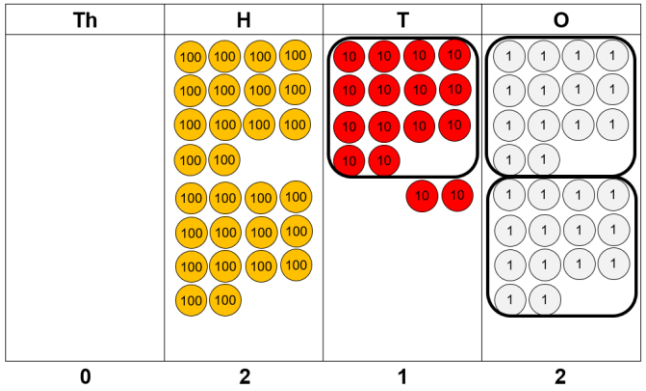


Pictorial

4. Exchange 1 hundred for 10 tens. There is 1 group of 14 in the tens column. There are 2 tens left over.



5. Exchange 2 tens for 20 ones. There are 2 groups of 14 in the ones column. There is no remainder.



Abstract

Formal written method:

÷ x - ↓

1. Divide
2. Multiply
3. Subtract
4. Pull down the digit

		0	2	1	2		1	4
1	4	2	9	6	8			
	-	0	↓			+	1	4
		2	9	↓			2	8
	-	2	8			+	1	4
		0	1	6			4	2
		-	1	4		+	1	4
			0	2	8		5	6
			-	2	8	+	1	4
					0		7	0

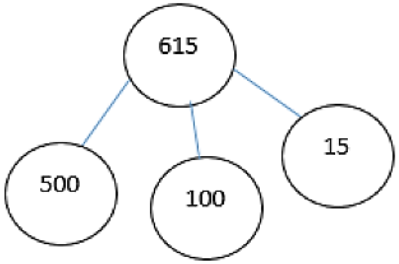
0 on top for the first digit – do not leave blank. Always finish the calculation at 0, even if there is no remainder. Answers can be expressed as a remainder, but this is unlikely in the Key Stage 2 SATS. Decimal answers given to no more than 2 decimal places.

Skip count the first five multiples of the divisor vertically, as shown. Continue to add to this as required.

	If dividing decimals: Multiply both the dividend and divisor by 10, 100 or 1000, as appropriate, so that they are integers. This will result in the same quotient. 320 ÷ 0.8 = 1. Multiply both the dividend and divisor by 10 so that both are integers: 3200 ÷ 8 = 2. We know that 32 ÷ 8 = 4, therefore 3200 ÷ 8 = 400. 320 ÷ 0.8 = 400 4.2 ÷ 0.07 = 1. Multiply both the dividend and divisor by 100 so that both are integers: 420 ÷ 7 = 2. We know that 42 ÷ 7 = 6, therefore 420 ÷ 7 = 60. 4.2 ÷ 0.07 = 60

Conceptual variation

Part whole model:
Using the part whole model below, how can you divide 615 by 5 without using short division?



Bar model:
150 ÷ 6 = ?

150					
?	?	?	?	?	?

Number sentence variations:

$\square = 2400 \div 6$

Children must understand that number sentences are always read from left to right.
Calculations should be written either side of the equality sign so that the sign is not just interpreted as the answer. “Something equals two thousand four hundred divided by six.”

Worded number sentences:
Divide 2400 by 6.
Share 2400 into 6.
How many groups of 6 are there in 2400?

Word problem:
I have £615 and I share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be split into 5 groups. How many pupils will be in each group?

Reasoning:
Rosie write 85 ÷ 3 = 28 r 1.
She says 85 must be 1 away from a multiple of 3. Do you agree?

Missing number problems:

$72 \div \square = 12$

$9876 \div \square = 6$

Use division to solve missing digit multiplication problems (using the inverse):

$3861 = \square \times 9$

$1496 = 8 \times \square$

$\square \times 4 = 3176$

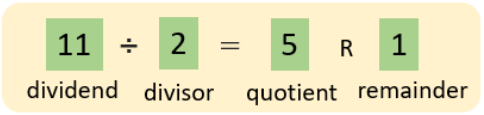
Key vocabulary

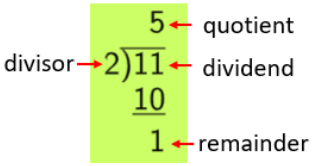
Children should then that unknown-factor problems can be represented with division equations (quotitive division), for example, 15 ÷ 5 = □.They should be able to use skip counting or their multiplication-table fluency to find the quotient: 15 ÷ 5 = 3.
Children should be able to describe how each term in the division equation links to the context and describe the division equation in terms of ‘division into groups’.

Language focus
“The 15 represents the total number of biscuits.”
“The 5 represents the number of biscuits in each bag.”
“The 3 represents the number of bags.”
“15 divided into groups of 5 is equal to 3.”

Children also need to be able to solve division calculations that are not set in contexts. They should recognise that they need to skip count in the divisor, or use the associated multiplication fact, to find the quotient. For example, to calculate 60 ÷ 10 = □, they can skip count in tens (counting the required number of tens) or apply the fact that 6 x 10 = 60.

Children should learn and use the correct names for the terms in division equations:





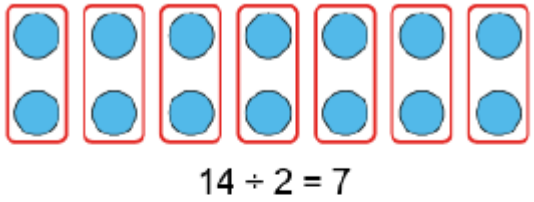
division, divide, divide by, divided into, divisible by, share, share equally, group, groups of, how many groups of ___ in ___ , one each, two each array, number line, left, left over, remaining, remainder, inverse, short division, long division, exchange, regroup, multiple, factor, quotient, divisor, dividend, prime number, prime factors, composite number (non-prime), common factor

Children will need to solve a range of different division problems using both equal grouping and equal sharing. It is especially important that children understand the concept of equal grouping as this is the concept applied to the formal method of short and long multiplication.

Equal grouping (quotitive division)

In an equal group problem, you know the number of objects each group should receive as well as the total number of objects you can distribute. The objective here is to find out how many (equal) groups can be created.

Example:
I need 14 ping-pong balls. There are 2 ping-pong balls in a pack. How many packs do I need?



14 ÷ 2 = 7

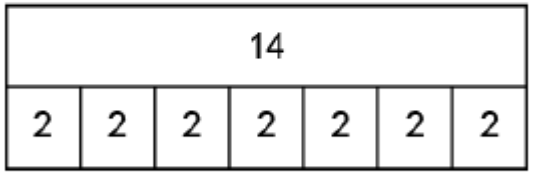


Figure 27: using an array and bar model to show that 14 divided into groups of 2 is equal to 7

Language focus:

"7 times 2 is 14, so 14 divided by 2 is 7."

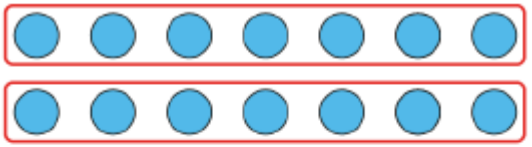
"14 divided into groups of 2 is equal to 7."

I need 7 packs of ping-pong balls.

Equal sharing (partitive division)

In equal sharing problems, you start with the number of groups and the total number of objects you want to share between them. Your goal is to find out how many objects each (equal) group can receive.

Example:
£14 is shared between 2 children. How much money does each child get?



14 ÷ 2 = 7

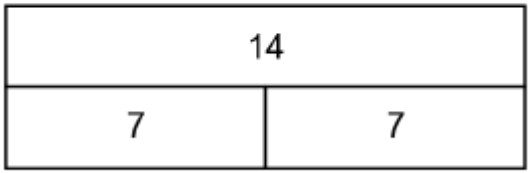


Figure 28: using an array and bar model to show that 14 shared between 2 is equal to 7

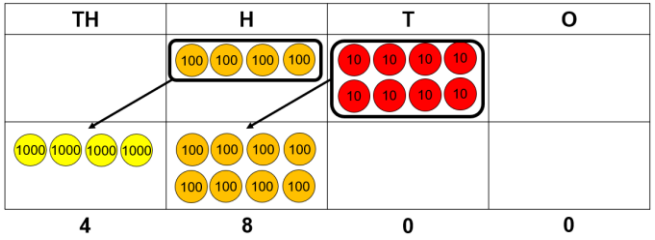
Language focus:

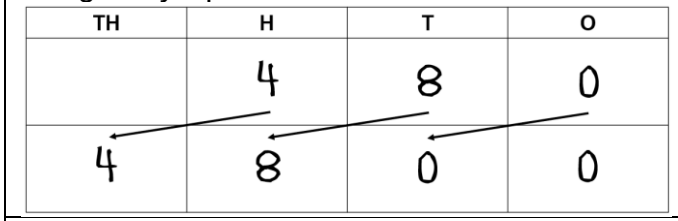
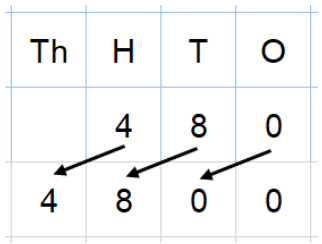
"7 times 2 is 14, so 14 divided by 2 is 7."

"£14 shared between 2 is equal to £7 each."

Each child receives £7.

Multiply and divide by 10, 100 and 1000

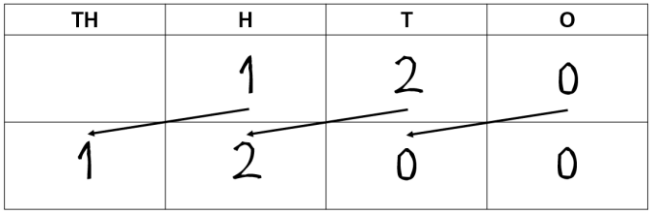
<table>
<tr><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Multiply by 10, 100 and 1000:
480 x 10 =
Physically move Dienes or counters showing the exchange (e.g. 8 tens for 8 hundreds and 4 hundreds for 4 thousands.
 

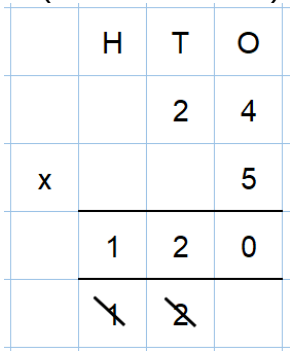
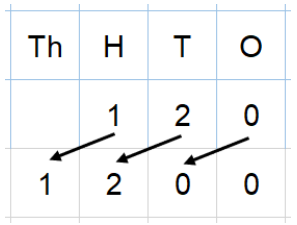
Children to physically move digits to the left using a drywipe board.
 
</td>
<td>Multiply by 10, 100 and 1000:
480 x 10 =
 

Move each digit and signify with an arrow. Rewrite underneath.</td>
<td>Multiply by 10, 100 and 1000 (mentally):
480 x 10 = 4800

Avoid misconceptions: we do <u>not</u> add a zero to the end of a number. 480 + 0 would still equal 480. Children need to apply their understanding of place value: in this example, the additional placeholder zero changes the place value of each digit by one place which means that each digit becomes ten times larger.</td>
</tr>
<tr>
<td>Multiply by a multiple of 10, 100 and 1000:
24 x 50 =

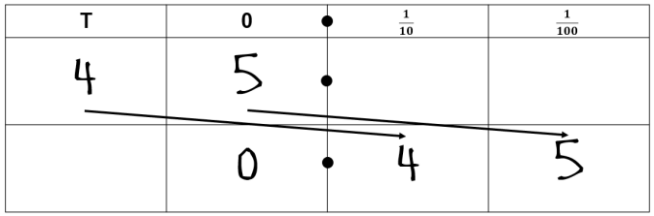
See column multiplication for concrete and pictorial representation of multiplication.

Physically move Dienes or counters showing the exchange (as above). Children to physically move digits to the left using a drywipe board.
 
</td>
<td>Multiply by a multiple of 10, 100 and 1000:
24 x 50 =

1. 24 x 5 = 120 (column method)
 
</td>
<td>2. 120 x 10 = 1200 (place value)
 

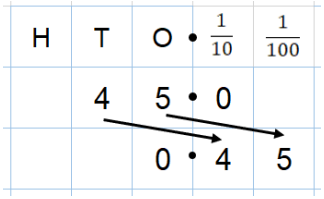
- Always perform two separate calculations
- Clear use of the column method for multiplication and use of place value for multiplying by 10, 100 or 1000</td>
</tr>
</table>

Divide by 10, 100 and 1000:
45 ÷ 100 =



Physically move digits to the right using a drywipe board.

Divide by 10, 100 and 1000:
45 ÷ 100 =



Move each digit and signify with an arrow. Rewrite underneath.

Divide by 10, 100 and 1000 (mentally):
45 ÷ 100 = 0.45

4500 ÷ 100 = 45

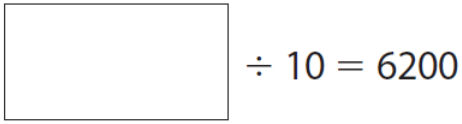
Avoid misconceptions: we do not take away zeroes from the end of a number. Children need to apply their understanding of place value: the place value of each digit changes.

Divide by a multiple of 10, 100 and 1000:
5600 ÷ 800 =

Physically move digits to the right using a drywipe board (as shown above).

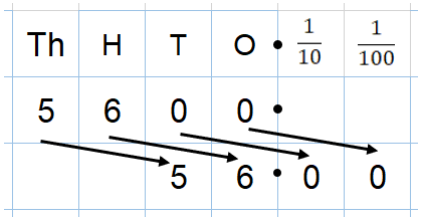
Children can use a multiplication square, or skip count, to support calculating multiplication and division facts.

Children can then use these facts to solve missing number problems:



Divide by a multiple of 10, 100 and 1000:
5600 ÷ 800 =

1. 5600 ÷ 100 =



2. 56 ÷ 8 = 7

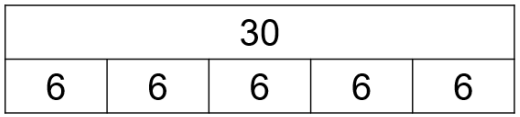
- If the final calculation is not a multiplication or division fact, use short or long division as required.

Developing fluency using the inverse operation

Children's understanding should go beyond the fact that multiplication and division are inverse operations. Children should be able to derive multiplication and division facts from arrays as well as from bar models.

3 x 6 = 18 18 ÷ 3 = 6

6 x 3 = 18 18 ÷ 6 = 3

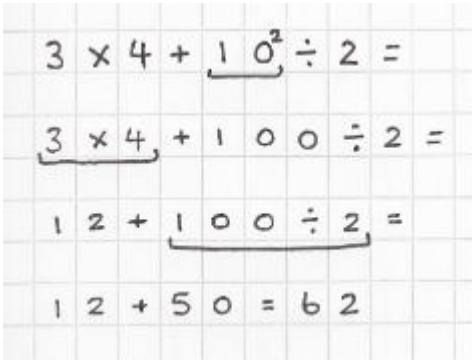


5 x 6 = 30 30 ÷ 5 = 6

6 x 5 = 30 30 ÷ 6 = 5

Calculation policy: additional guidance
Additional guidance for the modelling of key methods and concepts to ensure consistency across year groups.

BIDMAS

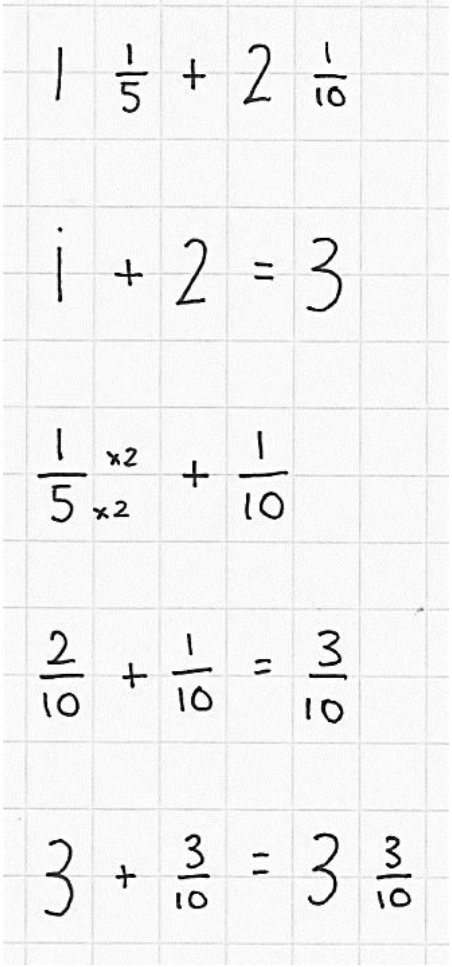
B Brackets **I** Indices **DM** Division and Multiplication **AS** Addition and subtraction Multiplication and division are performed whichever comes first in the number sentence from left to right. Addition and subtraction are performed whichever comes first in the number sentence from left to right.	**Steps to success:** 3 x 4 + 10^2 ÷ 2 =  1. Write the number sentence carefully. 2. Work through BIDMAS step by step. 3. Underline the part of the calculation you are solving. 4. Rewrite the whole calculation underneath each time.

Fractions: the four operations

Add mixed numbers

$$1\frac{1}{5} + 2\frac{1}{10} =$$

Steps to success:
1. Add the integers.
2. Add the fractions. Convert fractions so that they share a common denominator (as shown below).

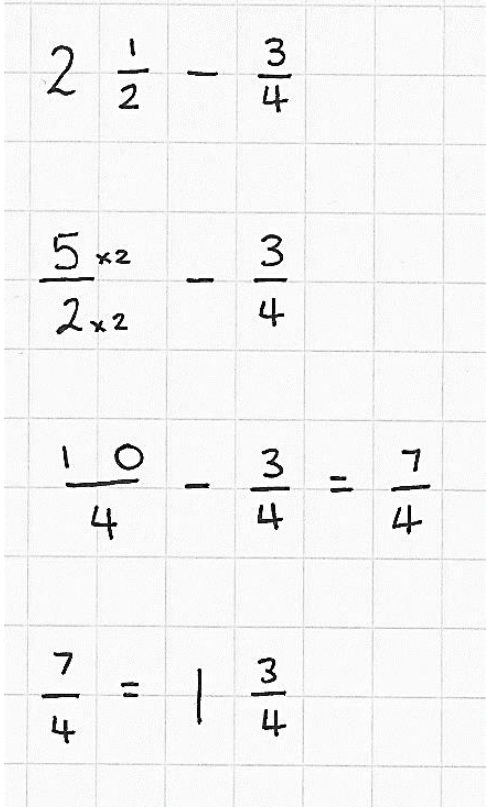


Subtract mixed numbers

$$2\frac{1}{2} - \frac{3}{4} =$$

Steps to success:
1. Convert any mixed numbers to improper fractions.
2. Convert fractions so that they share a common denominator (as shown below).
3. Subtract.

Ensure that children do <u>not</u> attempt to subtract any integers first. This can lead to further complications in the calculation.



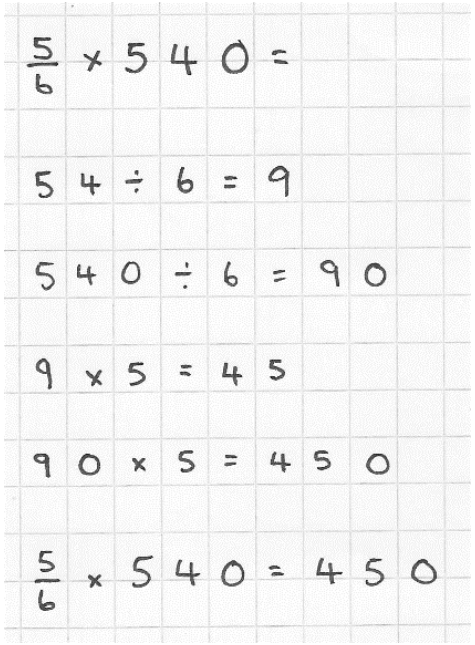
Children to complete any additional workings out to the side.

Note: it is perfectly acceptable for the children to leave their answer as an improper fraction, especially as errors can be made converting to a mixed number.

Multiply by fractions and mixed numbers

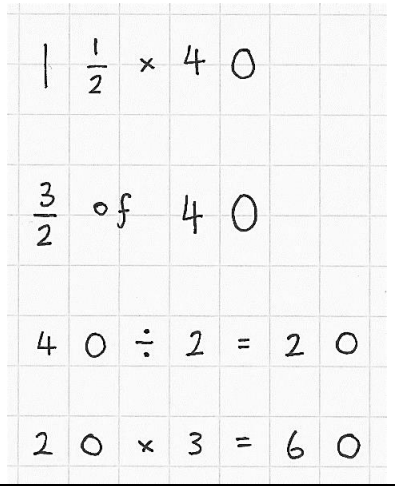
$\frac{5}{6} \times 540 =$ or $\frac{5}{6}$ of $540 =$

Steps to success:
1. Divide by the denominator.
2. Multiply by the numerator.



$1\frac{1}{2} \times 40 =$

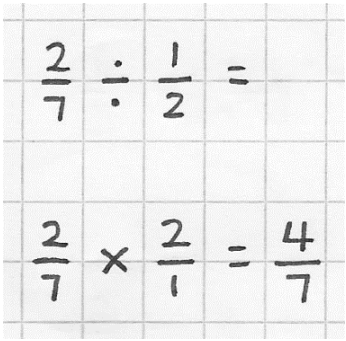
Steps to success:
Convert to an improper fraction and solve as above.



Divide by whole numbers and fractions

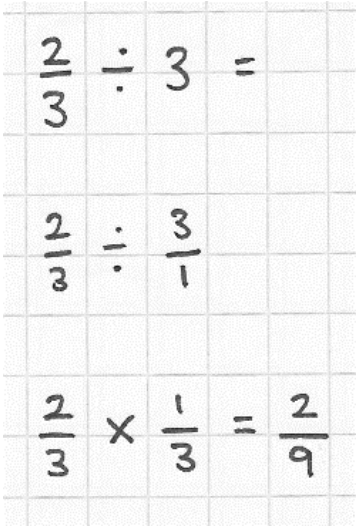
$\frac{2}{7} \div \frac{1}{2} =$

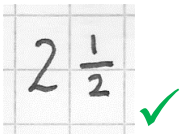
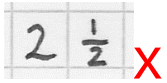
Steps to success:
1. Keep me.
2. Change me.
3. Flip me.



$\frac{2}{3} \div 3 =$

In this example, where the divisor is an integer, the children must first write the integer as a fraction over 1.



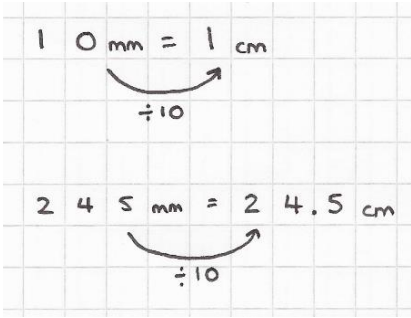
Conceptual variation	Presentation expectations:	Key vocabulary	
Balanced equations: $\frac{5}{7} - \frac{2}{7} = \frac{1}{7} + \square$	**Presentation expectations for mixed numbers and fractions:** Fractions should be written with one digit per square. The integer in a mixed number must be written in two squares, one above the other, to avoid any misconceptions.  	Children must understand that x (multiply) is the same as 'of'. This also applies to percentages. Both 'x' and 'of' should be used interchangeably to develop fluency.	numerator, denominator, unit fraction, non-unit fraction, compare and order, tenths, hundredths, equivalent fractions, equivalent decimals, integer, proper fractions, improper fractions, mixed numbers, percentages, simplify

Converting units of measurement

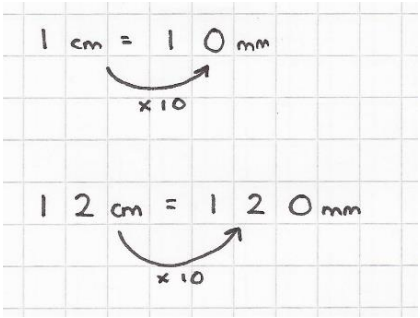
Steps to success:
1. Write what you know.
2. Show the conversion using an arrow.
3. Write what you are converting.
4. Show the conversion using an arrow.
3. Complete the calculation. Use place value to multiply and divide by 10, 100 and 1000.

Always show the conversion from **left to right** (not right to left). This reinforces the fact that equality statements are always read from left to right.

Convert 245mm into cm:



Convert 12cm to mm.:



Calculation policy: appendix

RUCSAC	Presentation expectations
RUCSAC steps to success for answering worded problems: **R** Read the question carefully. **U** Underline key information (or circle the operation). **C** Choose the correct method(s) and operation(s). **S** Solve the problem carefully. **A** Answer the question. Read the question again to ensure you have written the correct answer. **C** Check your answer. Recalculate or use the inverse. Children should be in the habit of underlining key information, exactly as they would when answering reading comprehension questions.	• One digit per square. • Two square margin. • Short date. • Learning objective. • Do now: • TT: • Must: • Should: • Could: • Challenge: • Plenary clearly marked if completed in books. • Children to divide page into two columns, using a ruler, if appropriate to the task. • When completing an arithmetic test, children should divide their page into two columns using a ruler. Children to complete the test in the left-hand column. Children should use the column on the right-hand side to recalculate or to check their answer using the inverse operation. • Written sentences, including reasoning, to use the correct spelling, punctuation and grammar. This should be corrected, where appropriate, using the school's marking and feedback policy. • Red arrow to show progress in the lesson where a child has been moved on by an adult. Children may draw a green arrow to show where they have chosen to move themselves on if given prior permission by the class teacher. • Gold starts awarded for excellent presentation. • Marking and feedback given by the class teacher in compliance with the school's marking and feedback policy.